Finance Workshop

“Time is money.”

Benjamin Franklin

Roadmap for Workshop

- Lump Sum Time Value of Money
- Perpetuities & Annuities (Annual)
- APRs versus Effective Annual Rates
- Other-than-annual Compounding with Lump Sums
- Other-than-annual Payment Streams
- Bond Terminology, Yields & Prices
- Stock Valuation

Introduction to Time Value of Money

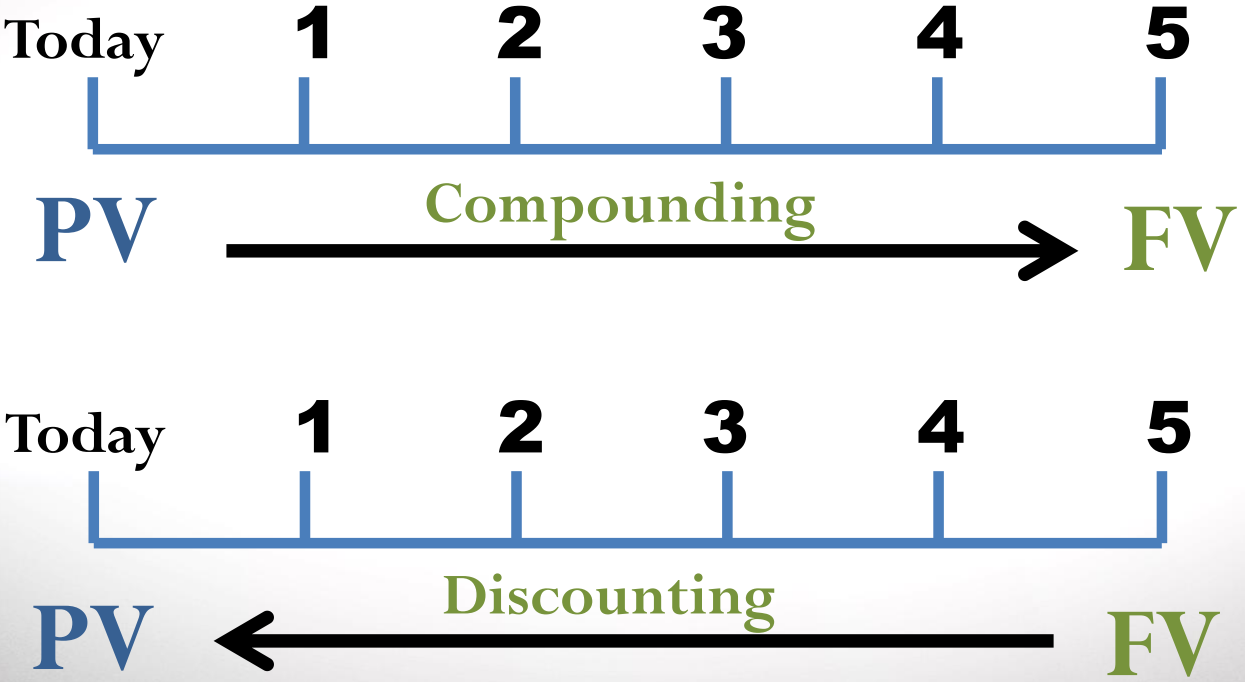
"Time Value of Money" is the general notion that a dollar today is worth more than a dollar later.

<u>Why</u> is this true?

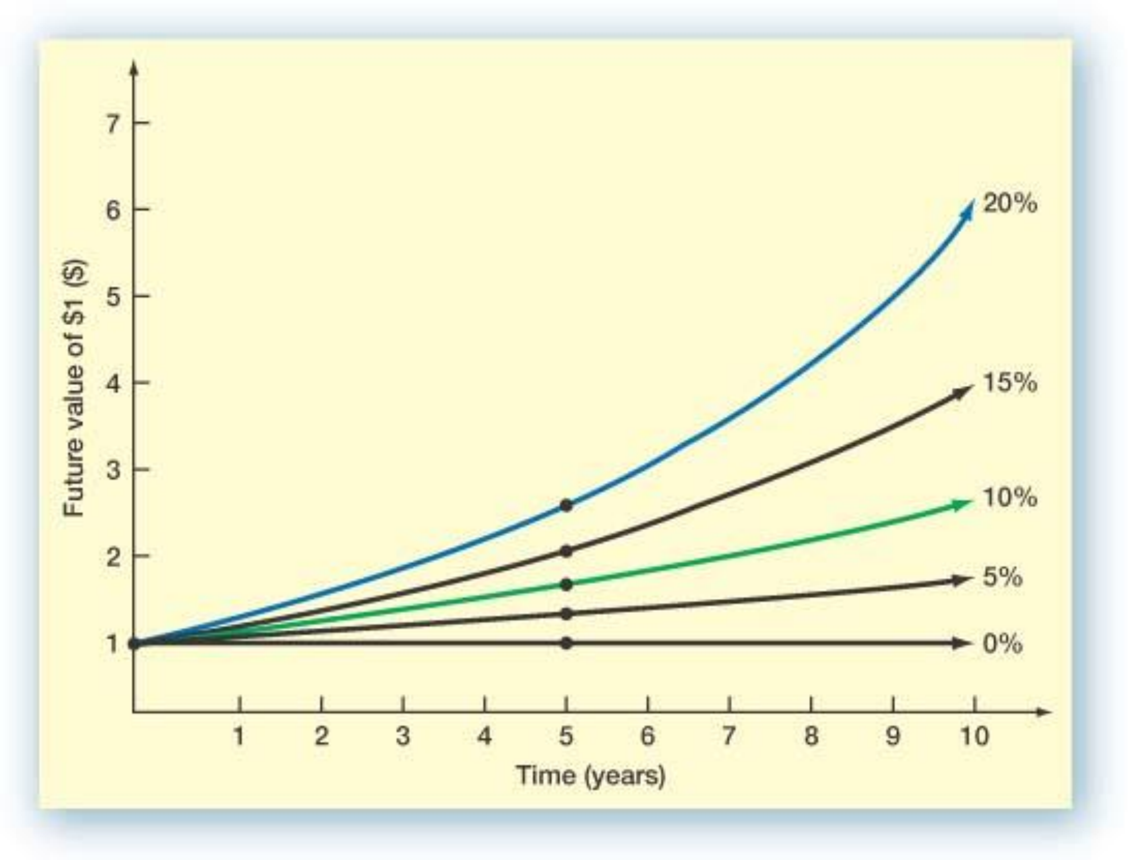
Definitions

- Present value = earlier money on a timeline
  - Price/value of an amount to be received in the future
- Future value = later money on a timeline
  - Amount an investment will grow to after one or more time periods
- Interest rate = “exchange rate” between earlier and later money
  - a.k.a. discount rate, cost of capital, opportunity cost, and required rate of return
  - Just as you can’t think of Canadian and US dollars as equivalent, you can’t think of earlier and later money as equivalent.

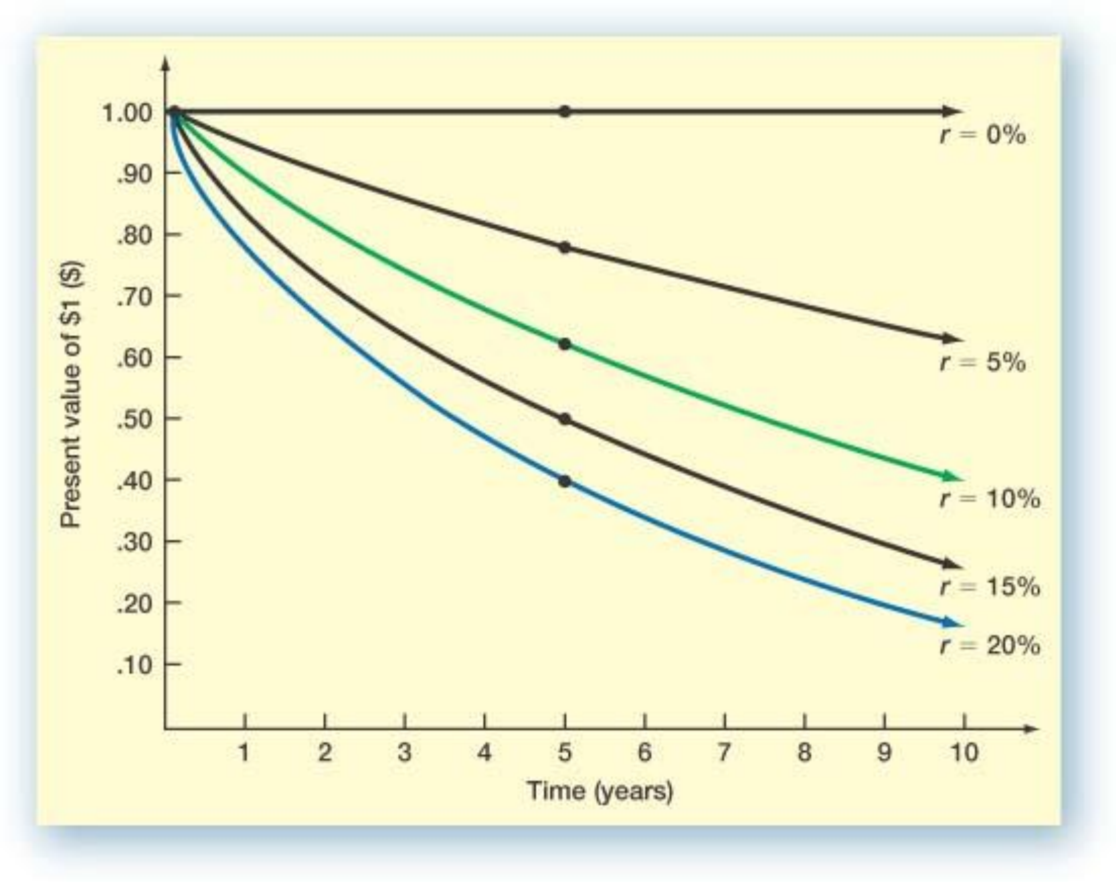
Finance uses "compounding" as the verb for going into the future and "discounting" as the verb to bring funds back to the present.



Future Values – increasing in time & interest rate



Present Values – decreasing in time & interest rate



Lump Sum Formula(s)

$$FV_n = PV(1+r)^n$$

$$PV = \frac{FV_n}{(1+r)^n}$$

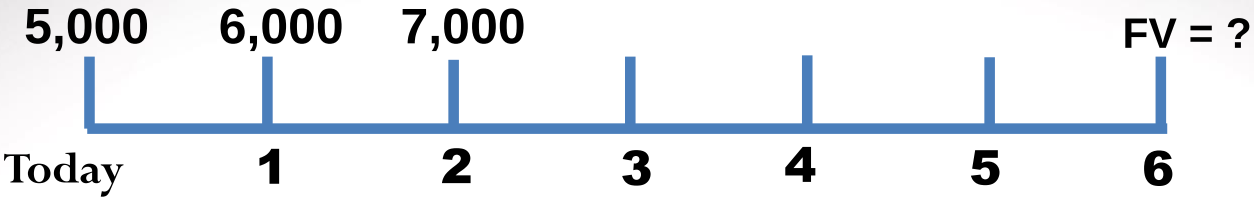
$$r = \left(\frac{FV_n}{PV}\right)^{1/n} - 1$$

$$n = \frac{\ln\left(\frac{FV_t}{PV}\right)}{\ln(1+r)}$$

Multiple Cash Flows (General Case)

- Treat each cash flow in the stream as a separate (lump sum) amount.
- Find the PV or FV of each individual cash flow.
- Then, simply add up the individual cash flow future or present values.
  - Note: Never add or subtract cash flows that don’t occur at the same point on the timeline.
  - And, never divide cash flows to spread them out in time or multiple them to aggregate them in time!

- With r = 4.5% : Given the following cash flows, what is the value of the investment at t=6?



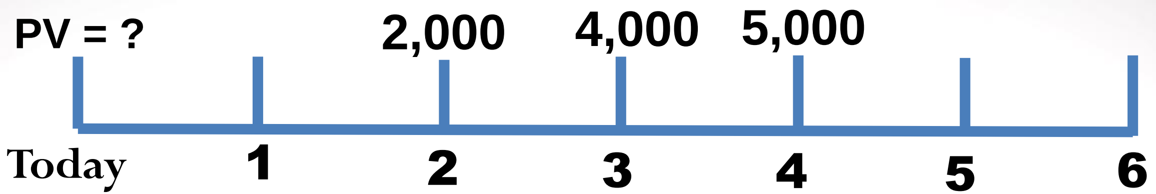
$$
\begin{aligned}
7{,}000(1.045)^4 &= \$\ \ 8{,}347.63 \\
6{,}000(1.045)^5 &= \quad 7{,}477.09 \\
5{,}000(1.045)^6 &= \underline{\quad 6{,}511.30} \\
& \quad \$\ 22{,}336.02
\end{aligned}
$$

Alternatively, \$5,000(1.045) = \$5,225 at t=1

(5,225 + 6,000)(1.045) = \$11,730.13 at t=2

$(11{,}730.13 + 7{,}000)(1.045)^4 = \$22{,}336.02$ at t=6

- With r = 6% : Given the following cash flows, what is the value of the investment today?



$1,779.99 = $2,000 ÷ $(1.06)^2$

3,358.48 = $4,000 ÷ $(1.06)^3$

<u>3,960.47</u> = $5,000 ÷ $(1.06)^4$

$9,098.94

Alternatively, $5,000/(1.06) = $4,716.98 at t=3

(4,716.98 + 4,000)/(1.06) = $8,223.57 at t=2

(8,223.57 + 2,000)/$(1.06)^2$ = $9,098.94

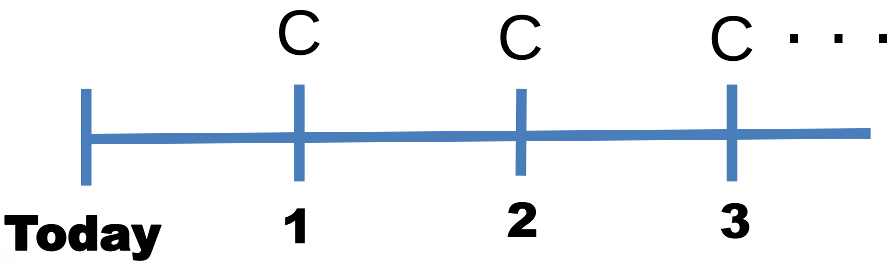
Things to Remember

- Even though we solved different versions of lump sum problems, ALL of this lesson’s time value of money (TVM) calculations use the SAME equation in four variables: PV, FV_n, r, and n
- Given any three of these input variables, you can always solve for the fourth.
- *Present* value and *future* value are defined *relative* to one another – they just mean *earlier* or *later* points in time.
  - *Present value* doesn’t have to be at time zero.
  - ALWAYS DRAW A TIME LINE!

Perpetuities

Definition and Timing

- Perpetuity = a stream of equal cash flows that occurs at regular intervals and lasts forever.



$$PV_\infty = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots$$

Perpetuity

$$PV_{\infty} = \frac{C}{r}$$

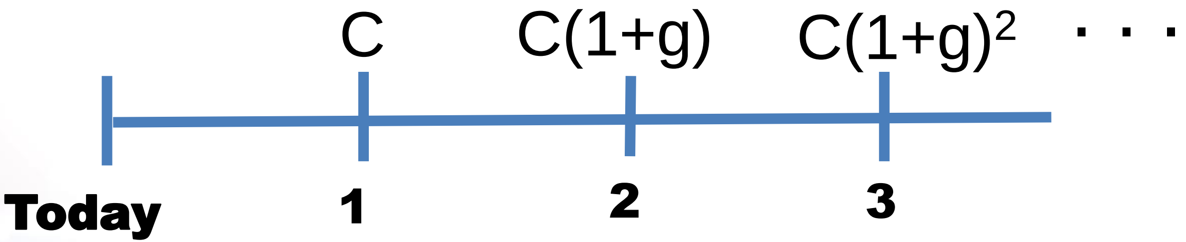
- NOTE: this formula is based on the assumption that the present value occurs *one period before* the first cash flow of the perpetuity.

- Thought question: How can an infinite stream of cash flows have finite value?

- Thought question: Is there any such thing as a perpetuity FV?

Growing Perpetuity

- Growing Perpetuity = a stream of cash flows that increases at a constant rate and occurs at regular intervals and lasts forever.



$$PV_{\infty} = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \ldots$$

Formula for Growing Perpetuity

- Growing perpetuity with first cash flow of C:

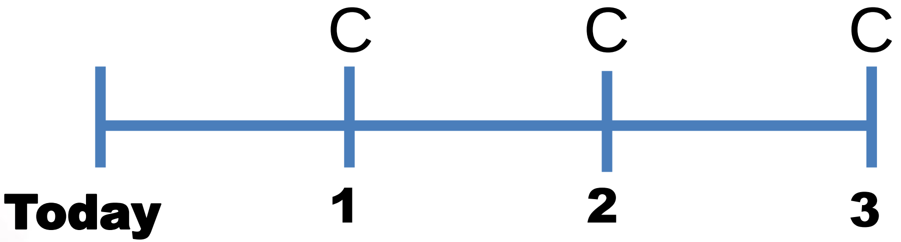
$$PV_{\infty} = \frac{C}{r-g}$$

  - Note that our previous perpetuity formula is a special case of this formula with g = 0.
  - Remember the timing of the cash flows! Make adjustments using the lump sum PV or FV calculations if necessary.

Annuities

Definition: Annuity

- Annuity = a stream of equal cash flows that occurs at regular intervals and ends after a specified number of payments.



N=3 in this example

$$AFV_3 = C(1+r)^2 + C(1+r) + C$$

$$APV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3}$$

Annuity Formulas

- The future value of an annuity of $C per period for N periods, at an interest rate of r is:

$$AFV_N = \frac{C}{r}\left[(1+r)^N - 1\right]$$

- The present value of an annuity of $C per period for N periods, at an interest rate of r is:

$$APV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

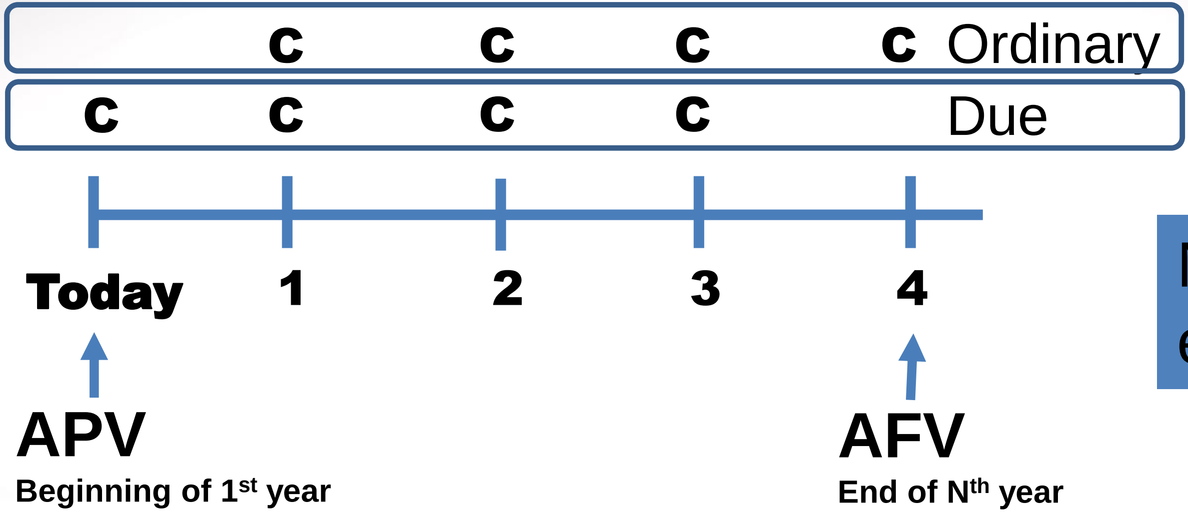
Timing of Annuity Payments

- Our *ordinary* formulas for APV and AFV implicitly assume *end*-of-period payments

- With an annuity *due*, the payments occur at the *beginning* of each period.

  - Note: The total number of payments is the same, it is just the timing of those payments that changes.

Ordinary Annuity versus Annuity Due



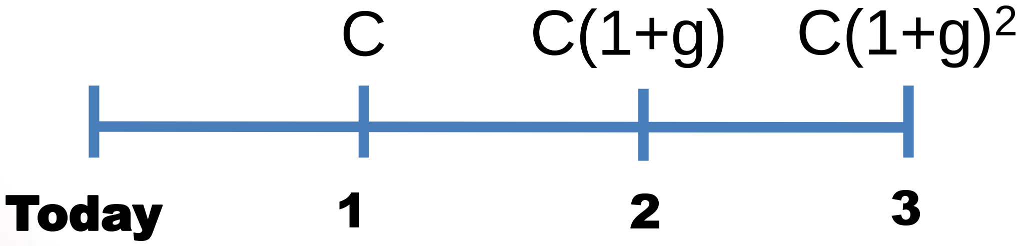
N=4 in this example

$$AFV_{due} = AFV_{ord}\,(1+r)$$

$$APV_{due} = APV_{ord}\,(1+r)$$

Definition: Growing Annuity

- Growing Annuity = a stream of cash flows that grows at a constant rate and occurs at regular intervals and ends after a specified number of payments.



N=3 in this example

$$APV = \frac{C}{(1+r)} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3}$$

$$AFV_3 = C(1+r)^2 + C(1+g)(1+r) + C(1+g)^2$$

Formulas for Growing Annuity

- Present value of a growing annuity with first cash flow of C and a total of N payments:

$$APV = \frac{C}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^{N}\right]$$

- Future value of a growing annuity with first cash flow of C and a total of N payments:

$$AFV = \frac{C}{r-g}\left[(1+r)^{N}-(1+g)^{N}\right]$$

Future & Present Value Formulas

Lump Sums

$$FV_n = PV(1+r)^n$$

$$PV = \frac{FV_n}{(1+r)^n}$$

Annuities

$$AFV = \frac{C}{r}\left[(1+r)^N - 1\right]$$

$$APV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

$$AFV_{due} = AFV_{ord}(1+r)$$

$$APV_{due} = APV_{ord}(1+r)$$

Perpetuities

$$PV_\infty = \frac{C}{r}$$

$$PV_\infty = \frac{C}{r-g}$$

Growing Annuities

$$AFV = \frac{C}{r-g}\left[(1+r)^N - (1+g)^N\right]$$

$$APV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^N\right]$$

*Note: Formulas in blue-shaded areas can be solved using financial calculator functions.

APRs versus Effective Annual Rates

Definitions

- *Effective annual rate (EAR)* = the rate, on an annual basis, that reflects compounding and tells the dollars you will actually have at the end of the year.
  - The text also calls this the *annual percentage yield (APY)*
- *Annual percentage rate (APR)* = the rate per period times the number of periods per year.
  - If the interest rate is 1.25% per month, the APR = 12(1.25) = 15% per year.

Annual Percentage Rate (APR)

This is the annual rate that is quoted by law on all loans.

By definition:

APR = periodic rate times the number of periods per year

Converting Interest Rates

- An *APR* that is compounded *m* times per year can be converted into an *EAR* using:

$$1 + EAR = \left(1 + \frac{APR}{m}\right)^m$$

The Effect of Compounding (APR=10%)

Compounding Period	Number of Times Compounded per Year	Effective Annual Rate (EAR)
Year	1	10.00000%
Quarter	4	10.38129%
Month	12	10.47131%
Week	52	10.50648%
Day	365	10.51558%
Hour	8,760	10.51703%
Minute	525,600	10.5170908%
Instantaneous	infinite	10.5170918%

Interest Rate Examples

– First National Bank charges 13.2 percent compounded monthly on its business loans. First United Bank charges 13.5 percent compounded semiannually. As a potential borrower, which bank would you go to for a new loan?

– Barcain Credit Corp. wants to earn an effective annual return on its consumer loans of 15 percent per year. The bank uses daily compounding on its loans. What interest rate is the bank required by law to report to potential borrowers? Explain why this rate is misleading to an uninformed borrower.

Things to Remember

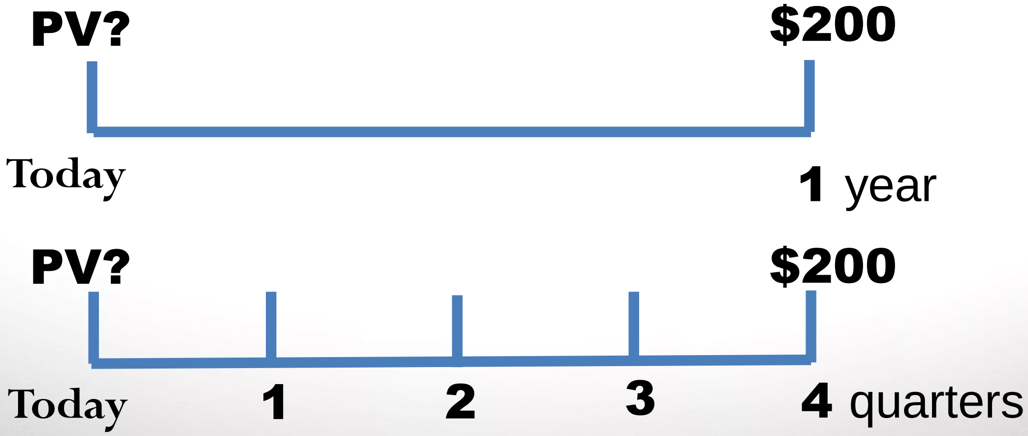
- Just as the temperature of a room can be described in either degrees Celsius or degrees Fahrenheit, an interest rate can be described as an APR or an EAR.
  - And, just as you don’t change the temperature of the room when you convert your description of that temperature from ^{o}C to ^{o}F (or vice versa), you don’t change the interest rate of an investment or account when you convert your description of it from APR to EAR (or vice versa).

- Given one description of the interest rate, you can ALWAYS convert it to a different description.
  - An account or investment has only ONE interest rate, but you can describe that rate in multiple ways.

Other-than-Annual Compounding with Lump Sums

Lump Sum Problems

- The timing of cash flows looks the same whether we count time in years or in “other” periods
  - Example: How much would you pay today for an investment that provides $200 one year from now?



Two Approaches

- Because the nature of the problem is identical, we can <u>choose</u> whether to count time in years or in “other” periods.
  - Both approaches will give us the SAME solution
- Method 1:
  - Express the interest rate as an EAR and use the number of years for *n*
- Method 2:
  - Express the interest rate as (APR/*m*) where *m* is the number of periods <u>per year</u> and use the number of periods (= *m* x number of years) for *n*

Things to Remember

You ALWAYS need to make the interest rate and the time period match.

- If you are looking at annual periods, you need an annual rate.
- If you are looking at monthly periods, you need a monthly rate.

Other-than-Annual Payment Streams

Things to Remember

- You still have to use an interest rate that matches the problem, but there's only one correct approach.
- Annuity/Perpetuity problems:
  - N is always the <u>number</u> of payments
  - MUST match interest rate to the frequency of the cash flows
  - For <u>annual</u> payments, use an EAR
  - For <u>other-than-annual</u> payments, use (APR/*m*) with APR compounding that matches the frequency of payments
- If the version of the interest rate given in the problem isn't what you need, simply convert it.

Future & Present Value Formulas

Lump Sums

$$FV_n = PV(1+r)^n$$

$$PV = \frac{FV_n}{(1+r)^n}$$

$$1 + EAR = \left(1 + \frac{APR}{m}\right)^m$$

Annuities

$$AFV = \frac{C}{r}\left[(1+r)^N - 1\right]$$

$$APV = \frac{C}{r}\left[1 - \frac{1}{(1+r)^N}\right]$$

$$AFV_{due} = AFV_{ord}(1+r)$$

$$APV_{due} = APV_{ord}(1+r)$$

Perpetuities

$$PV_\infty = \frac{C}{r}$$

$$PV_\infty = \frac{C}{r-g}$$

Growing Annuities

$$AFV = \frac{C}{r-g}\left[(1+r)^N - (1+g)^N\right]$$

$$APV = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^N\right]$$

*Note: Formulas in shaded areas can be solved using financial calculator functions.

Bond Terminology, Yields & Prices

*A bond is a **contract** between two parties: one is the **investor** and the other is an **issuer** (a company or government agency)*

Selling a bond = Borrowing money
Buying a bond = Lending money

A bond investor can either keep the bond until it matures and receive the promised payments over the life of the bond, or sell the bond to another investor in the secondary market.

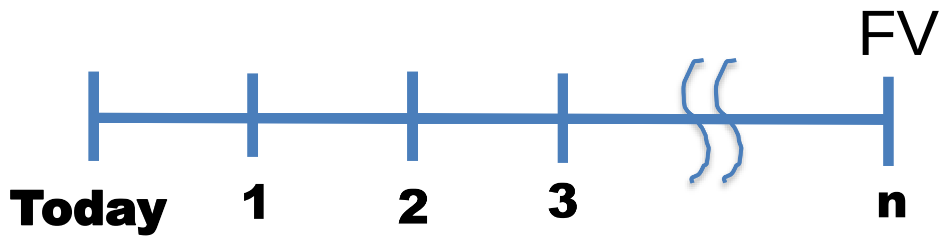
Important Bond Definitions

Terms
Certificate, or Indenture
Coupons
Coupon Rate
Face, or Par Value
Maturity Date
Term
Yield
Default

Descriptions/Features
a. Regular interest payments made every period until the bond matures.
b. Principal amount repaid to each bond owner when the bonds mature, usually $1000 per bond.
c. Date on which the principal is repaid.
d. Time remaining until the maturity date.
e. Failure to abide by all terms and conditions of the indenture (can be “financial” or “technical”).
f. Interest rate that describes the interest payments.
g. The legal contract that describes the terms and structure of the loan agreement.
h. The return that investors require to make them willing to buy the bond at its current price.

Bond Pricing: Zero-coupon Bonds

- The price of a bond is equal to the present value of its promised future cash flows.

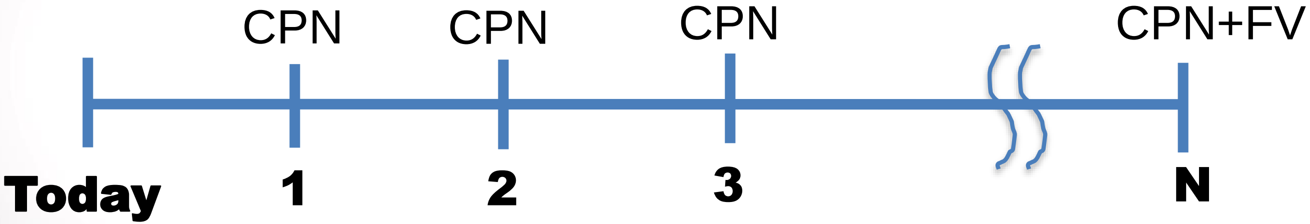


- *Zero-coupon (pure discount)* bonds: $P_{zero\ coupon} = \dfrac{FV}{(1+y)^n}$
  - Always sold at a discount to their face value. The return on this lump-sum future cash flow represents the interest received by the investor.

Bond Pricing: Coupon Bonds

- Periodic coupon payment:

$$CPN = \frac{Coupon\ Rate \times Face\ Value}{Number\ of\ Coupon\ Payments\ per\ Year}$$



- Coupon bond price:

$$P_{bond} = \underbrace{CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right)}_{\text{Annuity PV}} + \underbrace{\frac{FV}{(1+y)^N}}_{\text{Lump Sum PV}}$$

The “coupon rate” is NEVER used as a discount rate.

Student alert!

Once you have computed the periodic interest payment, you can ‘throw away’ the coupon rate. You need the dollar amount of the payment (CPN), not the coupon interest rate itself.

The bond “yield” is the discount rate. This is the return investors require to be willing to buy the bond.

Yield-to-Maturity

- Yield-to-Maturity (YTM) = return investors require = the discount rate that sets the present value of the promised bond payments equal to the current market price.
  - For an n-year zero-coupon bond:

$$YTM_n = \left(\frac{Face\ Value}{Price} \right)^{1/n} - 1$$

  - For a coupon bond, solve the pricing equation for the rate, y=YTM:

$$P_{bond} = CPN \times \frac{1}{y}\left(1 - \frac{1}{(1+y)^N}\right) + \frac{FV}{(1+y)^N}$$

Discounts & Premiums

- *Par bond* = bond that is selling for face value
- *Discount bond* = bond selling for less than face value
- *Premium bond* = bond selling for more than face value

  - If coupon rate > YTM, the bond will sell at a premium.
  - If coupon rate = YTM, the bond will sell at par.
  - If coupon rate < YTM, the bond will sell at a discount.

Stock Valuation

One-Year Stock Investment

- The timeline of cash flows:

- Price today = present value of dividend + expected selling price (both received at t=1):

$$P_0 = \frac{Div_1 + P_1}{1 + r_E}$$

r_E is the return available in the market on other investments of equivalent risk to this firm's stock

Total Return

- Solving for the discount rate, we see the two components of stock return:

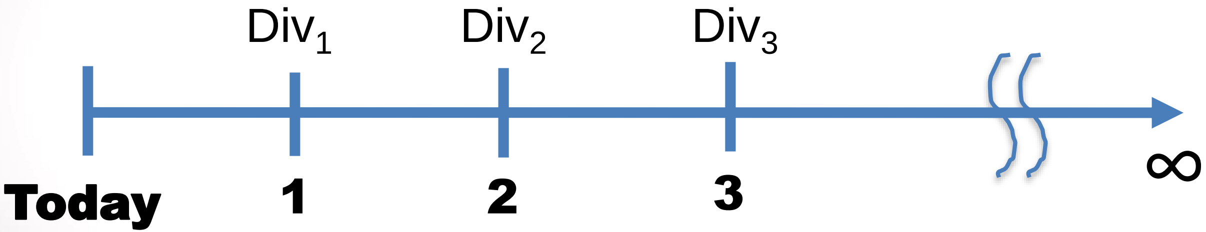
$$r_E = \underbrace{\frac{Div_1}{P_0}}_{\text{Dividend Yield}} + \underbrace{\frac{P_1 - P_0}{P_0}}_{\text{Capital Gain Rate}}$$

Total Expected Return

  - r_E is called the ***equity cost of capital*** (or, simply ***cost of equity***)

General Stock Valuation

- Just as the price of a bond equals the present value of its coupon payments and face value, stock price is the present value of all future dividend payments



$$P_0 = \frac{Div_1}{1+r_E} + \frac{Div_2}{(1+r_E)^2} + \frac{Div_3}{(1+r_E)^3} + \ldots$$

Note: Analogous to bond yield, r_E is the return investors require to make them willing to buy this stock.

Complications

- Unlike a bond that has a fixed maturity date, stock is an *infinite-lived* investment.
- Unlike a bond's contractually-defined coupon interest payments, dividends are set at the discretion of the firm and can *vary over time.*
  - This means that to value stock, we must make <u>assumptions</u> about the pattern of dividend payments we expect AND we have to be willing to assume that pattern will *eventually* become a simple perpetuity or growing perpetuity.

1) Constant Dividends (Zero Growth)

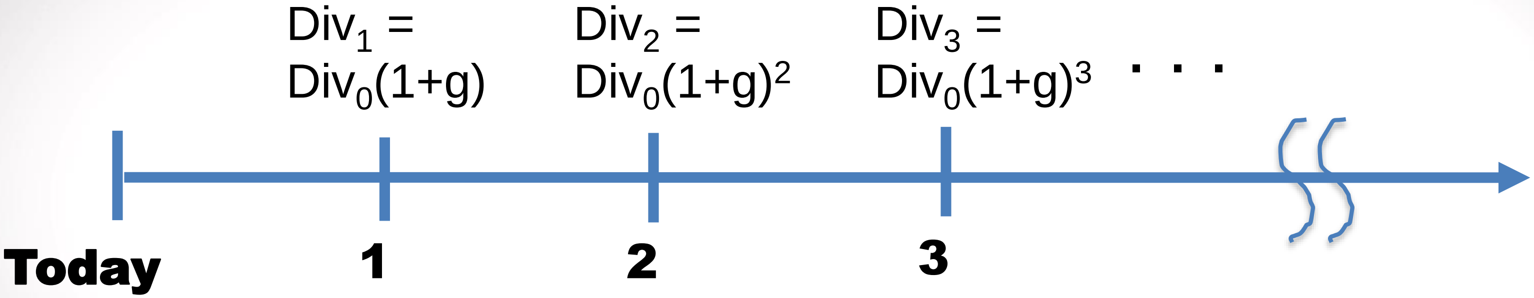
- We might assume that each year's dividend is the same, equal to a constant Div (e.g., preferred stock)
  - This is a perpetuity, so $P_0 = \dfrac{Div}{r_E}$
  - What happens one year from now?

$$P_1 = PV\ of\ remaining\ future\ dividends = \frac{Div}{r_E}$$

    - Note: $P_1 = P_0$, so NO capital gains under this assumption.

2) Constant Dividend Growth

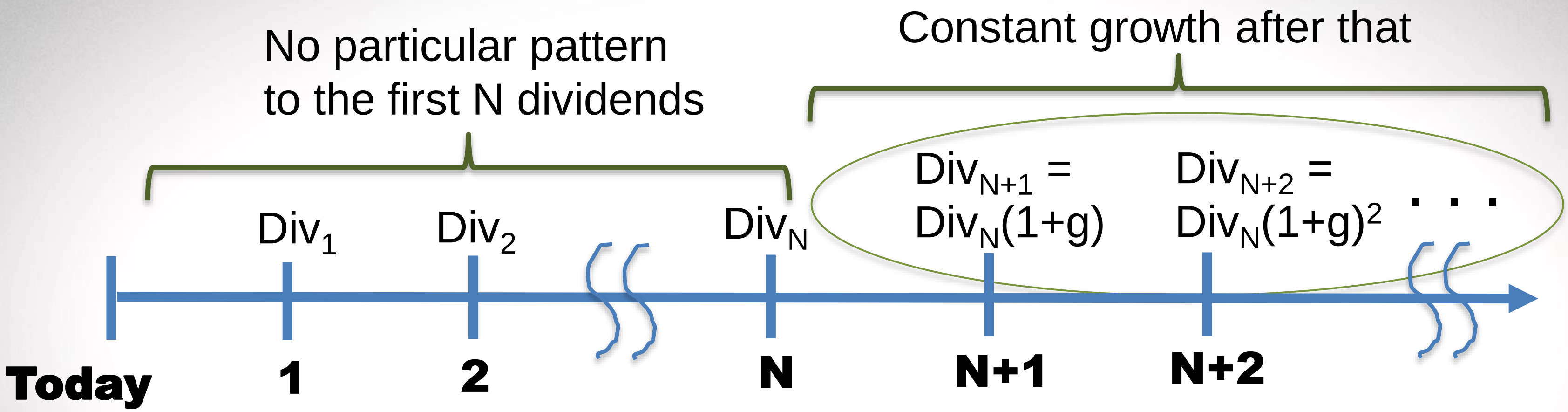
- We might assume that the firm just paid a dividend of Div_0 and dividends grow at a constant rate (g), then



  – This is a growing perpetuity with first cash flow of Div_1 so: $$P_0 = \frac{Div_1}{r_E - g} = \frac{Div_0(1+g)}{r_E - g}$$

  – Under this assumption, the capital gains rate = dividend growth rate, g, so: $$P_N = P_0(1+g)^N$$

3) Eventually Constant Dividend Growth



The circled part is a *growing perpetuity* with first cash flow of Div_{N+1}

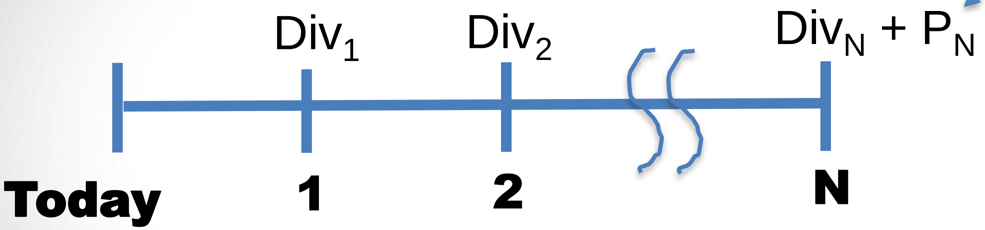
– Since the perpetuity formula gives us the value one period before the first cash flow: $P_N = \dfrac{Div_{N+1}}{r_E - g}$

Does NOT include the value of the first N dividends.

The PV at t = N of Div_{N+1}, Div_{N+2}, … etc

3) Eventually Constant Dividend Growth (cont.)

We can replace all the dividends from N+1 onward with their PV at t = N



The dividends are now an uneven (but finite!) cash flow stream, so discount them as individual lump sums to get the price of the stock today:

$$P_0 = \frac{Div_1}{1+r_E} + \frac{Div_2}{(1+r_E)^2} + \ldots + \frac{Div_N + P_N}{(1+r_E)^N}$$

Determinants of Growth

- Dividend growth is a function of the firm's earnings and how much of those earnings it retains/reinvests versus pays out as a dividend.
  - Payout Rate = (1 – Retention Rate)

- $Div_t = EPS_t \times Payout\ Rate$
  - Change in Earnings = New Investment x ROIC
  - New Investment = Earnings x Retention Rate
  - Earnings Growth Rate = (Change in Earnings/Earnings)
    = Retention Rate x ROIC

Profitable versus Unprofitable Growth

- Not all growth results in increased share price.
- Cutting a firm’s dividend to increase investment will raise the stock price if, and only if, the new investments have a positive NPV.
- NPV > 0 means ROIC > r_E

Stock Valuation Formulas

True in General

$$P_0 = \frac{Div_1}{1+r_E} + \frac{Div_2}{(1+r_E)^2} + \frac{Div_3}{(1+r_E)^3} + \ldots$$

$$r_E = \frac{Div_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

Constant Dividends (Zero Growth)

$$P_0 = \frac{Div}{r_E}$$

$$P_N = P_0$$

Constant Growth

$$P_0 = \frac{Div_1}{r_E - g} = \frac{Div_0(1+g)}{r_E - g}$$

$$P_N = P_0(1+g)^N$$

$$r_E = \frac{Div_1}{P_0} + g$$

Eventually Constant Growth

$$P_N = \frac{Div_{N+1}}{r_E - g}$$

$$P_0 = \frac{Div_1}{1+r_E} + \frac{Div_2}{(1+r_E)^2} + \ldots + \frac{Div_N + P_N}{(1+r_E)^N}$$